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Atmospheric gravity Green's functions

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THE observational accuracy of the superconducting gravimeter has now reached 0.1 μGal level or better, with characteristics of the low noise, high sensitivity and better continuity and stability. It will play an important part in the study of the change in (non-) tidal gravity signals, especially in the application of the geophysics and geodynamics. However, the instrument can simultaneously record atmospheric gravity signals due to the change in local and regional pressure when tidal gravity is measured. The influence of the atmospheric pressure on gravity usually takes three forms: (ⅰ) the direct attraction of the change in atmospheric masses, (ⅱ) the deformation of the elastic earth under the action of the atmospheric loads, and (ⅲ) the variable of potential of the earth due to mass redistribution in the interior of the earth. Farrell[1] introduced Green's functions in order to describe the response of the earth under the action of the mass load on surface of the earth; however, the problem is much more complex when the distribution of the mass density varies with its altitude. Therefore, we have to reconsider this problem. The past researches show that the contribution of the Newtonian term is mainly from the change in atmospheric masses in lower layers near gravity station, and the loading effect due to the deformation of the elastic earth on gravity is mainly from the change in atmospheric masses far away from station.

We studied the effect of the change in air pressure on gravity field using regional and global data sets for stations in Europe in the past years, and obtained satisfying results[2]. However, the serious disadvantages are that the computing model was relatively complex, needed a large expenditure due to a great number of data, and a lot of computing time. Some colleagues in earth science over the world tried to correct the influence of the air pressure on gravity using station pressure data, but the results are unsatisfying simply due to not considering the effect of the regional change in air pressure[3, 4]. Therefore, the main purpose of this study is to overcome the above-mentioned disadvantages, to introduce and to construct the atmospheric gravity Green's functions (AGGF) under the assumption of an SNREI (spherical, non-rotating, elastic and isotropic) earth model considering internationally used standard atmospheric models. Integrating these Green's functions, we can find a simpler and much more effective theoretical model in still keeping computing accuracy in order to obtain the correction value of the change in air pressure on gravity observations.

1 Newtonian attraction for a column model

The Newtonian attraction term $g_n(\psi)$ due to the change in atmospheric masses (assuming positive downward) can be directly computed by[5]

$$g_n(\psi) = -G\mathrm{d}\Omega\int_{\varepsilon}^{z_0}\rho(z)\left[\frac{(a+z)\cos\psi - a}{r^3}\right]\mathrm{d}z, \tag{1}$$

where G is the gravitational constant, $\mathrm{d}\Omega$ is an area of the column base expressed in steradians, r is the distance between the mass volume $\mathrm{d}v = \mathrm{d}\Omega\mathrm{d}z$ and the gravity station, a radius of the earth, ψ an arbitrary angular distance, z_0 represents a truncation height, ε is an initial integration value, $\rho(z)$ is atmospheric pressure density distributed with altitude. It must satisfy an ideal gas law:

$$\rho(z) = P(z)/[R \cdot T(z)], \tag{2}$$

where $P(z)$ and $T(z)$ are the air pressure and temperature in function of the altitude z, R is the ideal gas constant. In the case of an isothermal atmosphere, the pressure decreases exponentially with altitude

$$P(z) = P_0\exp(-z/\lambda), \tag{3}$$

where P_0 is the surface value, λ represents the atmospheric scale height. If the atmosphere is not isothermal, then the hydrostatic equations with observed temperature profile must be integrated over height, but the above equation is approximately correct for any height interval.

In order to consider various standard temperature models as a function of altitude, an analytic function is introduced as [6]

$$T(z) = T_0 + C_1\frac{z}{2} + \frac{1}{2}\sum_{i=1}^{n}\delta_i(C_{i+1} - C_i)\log\left[\frac{\cosh(z_i - z_i/\delta_i)}{\cosh(z_i/\delta_i)}\right], \tag{4}$$

where $C_{n+1} = 0$, $z_{i-1} < z < z_i$, T_0 represents the surface temperature, δ_i is the measured sharpness of the corner at z_i, C_i are the discontinuities at $\mathrm{d}T/\mathrm{d}z$.

From eq. (1), it can be seen that the Newtonian attraction of a column air model depends on the change in pressure and temperature at its base (see eqs. (3) and (4)). As an column area expressed in steradians is taken, consequently, the attraction of this column air can be interpreted as Green's functions. In order to distinguish these column air (distributed) Green's functions from the more familiar point mass (concentrated) loading Green's functions given by Farrell (1972), we call them here the atmospheric gravity Green's functions (AGGF). Together with the effect of the elastic deformation of the earth, integrating these Green's functions over an considered area around gravity station, the gravity signals due to the change in atmospheric pressure can then be simply obtained.

In the case of the thin atmosphere approximation, integration (1) can be transformed into

$$g_n(\psi) = \left[\frac{-g}{4m_e\sin(\psi/2)}\right]\mathrm{d}\Omega\int_{\varepsilon}^{z_0}\frac{P(z)}{R \cdot T(z)}\mathrm{d}z, \tag{5}$$

where m_e is the mass of the earth. Notice that the term inside the square brackets has the same form as that in the case of ocean loading given by Farrell, the difference is only the supplementary term inside integration. This term corresponds to the integration of the column air density with unit spherical area, from the earth's surface to a truncation height z_0. It represents clearly the dependence of the AGGF on the altitude which is the key difference compared with the surface mass loading.

Equation (1) can be transformed into the form in discrete summation, the numerical integration can then be done. In order to normalize the results, a normalization factor ($10^5 \cdot \psi/P_0$) is introduced, therefore the normalized AGGF values will be

$$GN(\psi) = [10^5 \cdot \psi] \cdot g_n(\psi)/P_0 \qquad (\mu\mathrm{Gal/hPa}). \tag{6}$$

The corresponding computing results of the AGGF values are given in fig. 1 (curve 1).

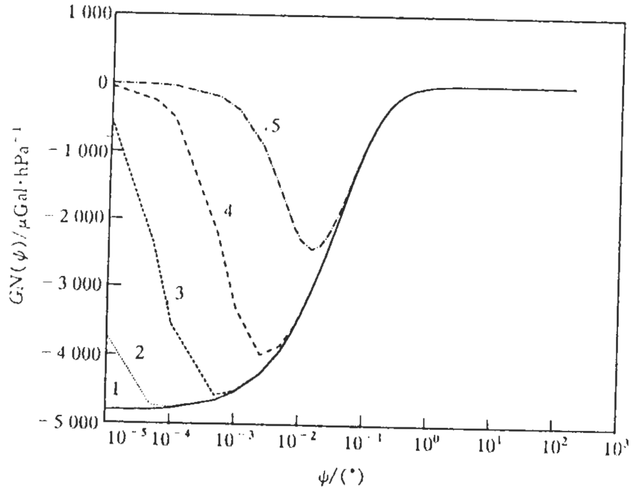

Fig. 1. Atmospheric gravity Green's functions $GN(\psi)$ values. 1, The results when the column base is located at the sea level; 2—5, results when the column base is taken as 0.1, 1.0, 10.0, 100.0 m above the sea level respectively. The scaling factor ($10^5 \cdot \psi / P_0$) is introduced in the calculation (see formula (6)), the constant term absorbed in the results includes the area of a spherical cap with radius 1°.

It is found that the AGGF values are relatively insensitive to the details of various atmosphere models. However, when introducing the two extreme specific gas constants R in the case of dry air and of the vapour water saturated model, a discrepancy of about 10% is found. In order to meet the general condition, the results given in this note are the average ones of the above two cases. From the numerical results, it is found that a minimum truncation height of 40 km is necessary for keeping computing accuracy. For example, an accuracy of 1% for the AGGF values at all the angular distance ψ can be assured when taking integration truncation height of 60 km.

2 Modification of the AGGF values

The constructed model mentioned above is based on the sea level only, as most of gravimeters are located on the continents above sea level, the effect of the station has to be considered when the AGGF results are to be applied. In order to solve this problem, we must consider a Taylor expansion around a zero height and consequently, we need the derivative of the $GN(\psi)$ with respect to the station height.

After considering station height δh , the first derivative of the column AGGF with respect to δh is then computed directly by

$$g'_n(\psi, \delta h)\mid_{\delta h=0} = -\frac{G\mathrm{d}\Omega}{2a^2\psi}\frac{P_0}{T_0R}\int_\varepsilon^{z_0}\frac{T_0 2a^2\psi}{P_0}\frac{P(z)}{T(z)}\left[\frac{J_{\mathrm{I}}}{r_h^5}\right]\mathrm{d}z, \tag{7}$$

where

$$r_h = [(a+z)^2 + (a+\delta h)^2 - 2(a+z)(a+\delta h)\cos\psi]^{1/2},$$
$$J_{\mathrm{I}} = (a+z)^2(1-3\cos^2\psi) - 2(a+\delta h)^2 + 4(a+\delta h)(a+z)\cos\psi.$$

The factor ($T_0 2a^2\psi / P_0$) in integration (7) is introduced in order to simplify the numerical computation. Owing to the properties of the non-linearity for the pressure and temperature distribution with altitude, truncation is not allowed after the first derivative of AGGF values with respect to δh . Therefore, we have to compute the second derivative of the AGGF values to station height with the following formula:

$$g''_n(\psi, \delta h)\mid_{\delta h^2=0} = \frac{G\mathrm{d}\Omega}{2a^2\psi}\frac{P_0}{T_0R}\int_\varepsilon^{z_0}\frac{T_0 2a^2\psi}{P_0}\frac{P(z)}{T(z)}(K_{\mathrm{I}} + K_{\mathrm{II}})\mathrm{d}z, \tag{8}$$

where

$$K_{\mathrm{I}} = [-4(a+\delta h) + 4(a+z)\cos\psi]/r_h^5,$$
$$K_{\mathrm{II}} = -5J_{\mathrm{I}}J_{\mathrm{II}}/r_h^7,$$
$$J_{\mathrm{II}} = (a+\delta h) - (a+z)\cos\psi.$$

The results obtained from eqs. (7) and (8) are normalized in the same way as those in $GN(\psi)$ given in expression (6) in the numerical integration, and are shown in figs. 2 and 3 (curve 1). From the figures, it can be seen that there exists a sharp decrease when the angular distance ranges from 0.01° to 0.1°, followed by an sharp increase when the angular distance ranges from 0.1° to 1.0°. For the second derivative of $GN(\psi)$ values, the total magnitude is relatively small and there exists a sharp decrease when the angular distance ranges from 0.01° to 1.0°, so that it tends to zero when the angular distance is larger than 1.0°. This indicates that the pressure effect from an area near gravity station is mostly geometric, and the effect from an area at large distance is mostly related with the change in total atmospheric masses.

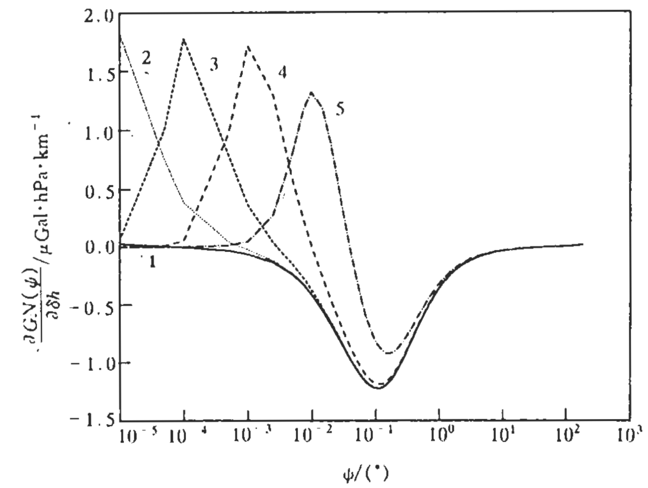

Fig. 2. The first derivative of the AGGF values with respect to station height $\partial GN(\psi)/\partial\delta h$. 1, The results when the column base is located at the sea level; 2—5, results when the column base is taken as 0.1, 1.0, 10.0, 100.0 m above the sea level respectively. The scaling factor ($10^5\cdot\psi/P_0$) is introduced in the calculation, the constant term absorbed in the results includes the area of a spherical cap with radius 1°.

In addition to the effect of the station height mentioned above, the topography around gravity station should not be neglected. The instruments are installed not only on a hill, but also on a valley or at stations below sea level (e. g. those in the Netherlands), the local ground fluctuations occupy part of atmospheric masses, the change of these masses can increase gravity. Assuming a column base uplift Δh, the integration of the AGGF values should be taken from a height Δh till the truncation height of the atmosphere above the earth's surface. The effect of the various column base heights on the AGGF values are computed, the results are given in fig. 1 (curves 2—5). From the analysis, it is found that the larger the height of the column base is, the larger the influence on the AGGF values and the wider the range around station will be. For

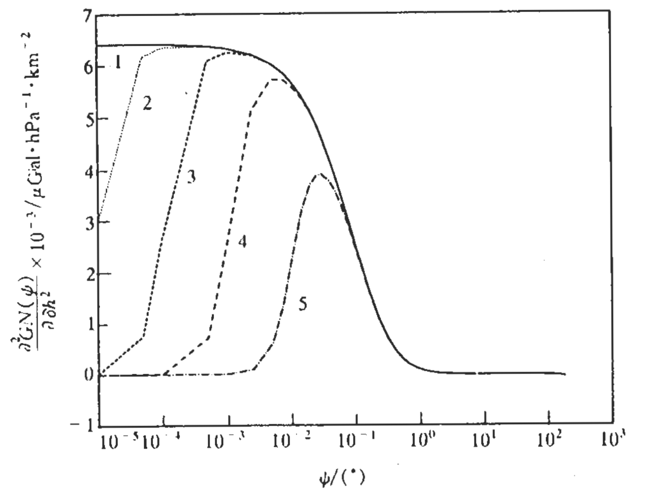

Fig. 3. The second derivative of the AGGF values with respect to station height $\partial^2 GN(\psi)/\partial\delta h^2$. 1, The results when the column base is located at the sea level; 2—5, results when the column base is taken as 0.1, 1.0, 10.0, 100.0 m above the sea level respectively. The scaling factor ($10^5\cdot\psi/P_0$) is introduced in the calculation, the constant term absorbed in the results includes the area of a spherical cap with radius 1°.

example, when Δh is taken as 10 and 100 m, the effect on the AGGF values can reach an area with an angular distance of 0.01° and 0.05° respectively. The effect of the various column base heights on the first and second derivatives of the AGGF values with respect to station height is also calculated, the corresponding results are given in figs. 2 and 3 (curves 2—5). The results also show that if the topography around gravity station within in local zone fluctuates by no more than 1 km, then the influence on gravity is of the order of some tens of nGal level.

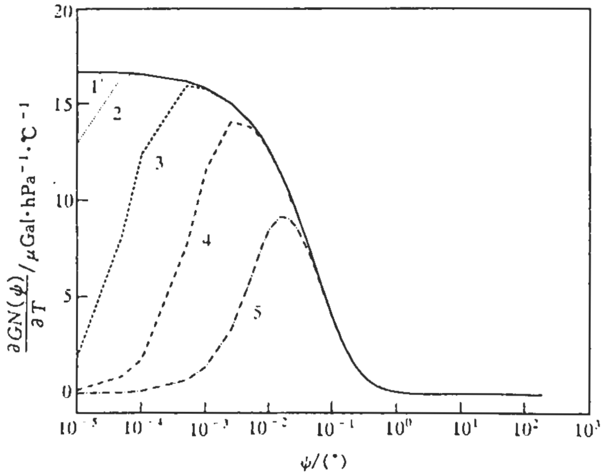

Fig. 4. The first derivative of the AGGF values with respect to temperature $\partial GN(\psi)/\partial T$. 1, The results when the column base locates at the sea level; 2—5, results when the column base is taken as 0.1, 1.0, 10.0, 100.0 m above the sea level respectively. The scaling factor ($10^5 \cdot \psi/P_0$) is introduced in the calculation, the constant term absorbed in the results includes the area of a spherical cap with radius 1°.

Since the temperature structure in the atmospheric layers at various altitudes depends on its surface value, an increase in temperature raises the center of the masses for a considered column air, but the total mass is unchanged. In order to calculate the effect of change in surface temperature on gravity, the first derivative of the column AGGF values with respect to temperature is calculated. In the region near the station, the upward pull of a column of air will be reduced by an increase in temperature, i.e. a reduction in the upward attraction is like an increase in downward attraction. Therefore, the first derivative of the AGGF with respect to temperature will be positive. The corresponding results are given in fig. 4 (curve 1). It is also found that the gravity signals due to change in temperature are relatively small. The influence of the various column base heights on the first derivative of the AGGF values to temperature is also computed (curves 2—5 in figure 4).

3 Results and discussion

After considering the topography and temperature effects, the final AGGF due to change in atmospheric pressure can be obtained based on the following formula:

$$GN(\psi) = GN(\psi)|_{table} + GN(\psi)|_T + GN(\psi)|_{\delta h} + GN(\psi)|_{\delta h^2}, \tag{9}$$

where $GN(\psi)|_{table}$ is the results computed directly (curve 1 in fig. 1), $GN(\psi)|_T$ is the term relating with the temperature correction, $GN(\psi)|_{\delta h}$ and $GN(\psi)_{\delta h^2}$ correspond to the correction of the station height, they are given as

$$GN(\psi)|_T = \frac{\partial GN(\psi)}{\partial T}(T_0 - 15℃),$$

$$GN(\psi)|_{\delta h} = \frac{\partial GN(\psi)}{\partial \delta h}\left[\frac{\delta h}{a\psi}\right],$$

$$GN(\psi)|_{\delta h^2} = \frac{\partial^2 GN(\psi)}{\partial \delta h^2}\left[\frac{\delta h}{a\psi}\right]^2.$$

The deformation of the elastic earth induced by the atmospheric mass loads is relatively small compared with the Newtonian attraction term and can be considered as the mass loads concentrated on the earth's surface. It is found that they are influenced mainly by a large area away from gravity station. Based on solving the dynamical equations of the free oscillation of the earth using a given earth's model as the PREM, the mass loading Green's functions can be theoretically computed. We calculated the response of the earth under the action of the point mass load for various earth's models and obtained the mass loading gravity Green's functions. The results show that the contribution of the details of the various earth models are relatively small. For example, if the uncertainty in earth models is about 10%, then the influence on gravity signals is in the order of some tens of nGal level. The detail procedures and the corresponding numerical results can be found in reference [2].

Using $g_e(\psi)$ to express the mass loading Green's functions of the atmosphere concentrated on the earth's surface, scaling them in the same way as that used in the column AGGF, and using $GE(\psi)$ to express the normalized values, we have

$$GE(\psi) = [10^5 \cdot \psi] \cdot g_e(\psi)/P_0 \qquad (\mu\text{Gal/hPa}). \tag{10}$$

When adding the direct effect due to the change in atmospheric masses and the effect of the elastic deformation of the earth due to the atmospheric mass loads, the gravity signals at an arbitrary angular distance can be computed as follows:

$$g(\psi) = \frac{GN(\psi) + GE(\psi)}{10^5 \cdot \psi} \frac{d\Omega \cdot P_0}{2\pi[1 - \cos(1^\circ)]} \qquad (\mu\text{Gal/hPa}). \tag{11}$$

After considering the topography and temperature correction as well as the contribution of the elastic deformation, the results, when integrating the above AGGF values on an area, indicate that the change in atmospheric pressure near station is the main contributor to gravity signals. The gravity signals obtained in a local zone with an angular distance of 0.5° at the station occupy 90% of the total gravity signals induced from global change in pressure, the gravity signals are relatively small for a regional zone between 0.5° and 10.0°, and are the same order as that obtained from a global zone for an angular distance greater than 10.0°. The behaviour of the atmospheric gravity signals, the coherence scale of the pressure, the time and the spatial scales appropriate to the hydrostatic approximation, and the distance of the station to the oceans, suggest us to have a division of the globe into local, regional and global zones.

From the numerical results, it is found that the AGGF values trend to zero at an angular distance of 2.75° where the Newtonian attraction changes its signs (curve 1 in fig. 1). It provides us probably with a proof to distinguish between mass load (as ocean loading) concentrated at the surface of the earth and a load (as atmosphere) distributed with function of altitude. On the other hand, the sum of the $GN(\psi)$ and $GE(\psi)$ has zero value at an angular distance near 1.0°. This indicates the existence of a rather broad band a few degrees from gravity station where the atmosphere makes no net contribution to gravity due to the opposite signs between them.

Integrating gravity effect at an area of an angular distance of 0.5°, we obtain an atmospheric gravity admittance of −0.360 3 μGal/hPa. This admittance implies that the contribution of the air pressure to gravity will be about 18 μGal for a year around pressure change of about 50 hPa at station. The results from tidal analysis based on the superconducting gravimeter from 1985 to 1994 at Wuhan station show that the response coefficient of gravity to station air pressure is −0.383 8 μGal/hPa for long period waves, and is −0.300 3 μGal/hPa for diurnal wave band. The results also correspond to the admittances at station Brussels obtained in our previous studies, i.e. -0.395 and -0.333 μGal/hPa for local and regional zones respectively when using ECMWF pressure data sets in Europe and a direct convolution method. In comparing tidal gravity observations obtained at stations of Wuhan (China) and of Brussels (Belgium), it is found that the observed tidal gravity residuals are mainly induced by the change in atmospheric pressure. The regional zone extends from an angular distance of 0.5° to 10° from gravity station. This is of the order of the synoptic scale of the weather systems, so that the pressure will generally correlate from one front to another thoughout this zone, but the correlation will weaken as distances from stations increase. Therefore, a net of the barometric stations is required from this zone in order to obtain the sufficient pressure signals. In this region, the obtained atmospheric gravity admittance is given as 0.059 μGal/hPa.

Atmospheric gravity signals depend on the change in surface temperature. The rising and falling of the surface temperature induce the expansion and contraction of the atmosphere. As the atmosphere warms, the center of mass of the local column of air moves further from gravity station, and the upwards attraction of the column diminishes. Therefore, in the vicinity of station, the sign of the atmospheric gravity admittance due to the change in temperature will be positive. Most of regional zone is below the instrument horizon, so its Newtonian attraction is positive and again diminishes as the center of the mass ascends. The regional zone temperature admittance is negative, and partially cancels that of local zone. The derivative of the AGGF values with respect to temperature trends to zero when the angular distance is greater than 2°. Therefore, the gravity effect due to temperature change can be neglected for a region of angular distance greater than 10°. When combining the temperature effect for both local and regional zones, the temperature admittance will be $0.014(T_0 - 15°)$ μGal/℃.

In the global zone ($\psi > 10°$), the main contributor to station gravity signals is the total column mass, not the atmospheric mass distribution throughout a column of air. The studies show that most of power of the signals pressure change is concentrated in a period of several days to weeks. On the other hand, we have to consider the effect of the inverted barometer ocean since most of the region is located on ocean area when angular distance is greater than 10°. Wunsch (1972) pointed out that in period between 40 and 400 h, the sea surface response to air pressure is as an inverse barometer[7]. In our previous study, we have considered the two extreme cases: (i) the inverted barometer ocean (IBO) and (ii) the non-inverted barometer ocean (NIBO), when the effect of the pressure on gravity and on surface displacements for various stations in Europe are calculated using regional atmospheric pressure data sets. The results show that for stations near the coastal line, a disparity of about 20% can be found when using the above two different models and it is also found that it fits much more to the real condition in the case of an IBO model. We obtained relatively ideal results when the observed tidal gravity data are corrected in using theoretical atmospheric gravity signals. It is found that most of the observed tidal gravity residuals are influenced by air pressure, and specially, it is much more effective in the period of several days and weeks.

4 Conclusions

The AGGF are constructed and introduced under the assumption of an SNREI earth model. Considering the internationally used standard atmosphere models, the column AGGF values in an arbitrary angular distance are calculated using a discrete integration method. Integrating these functions around gravity station when adding the effect of the elastic deformation of the earth due to the mass loads, it is found that the atmospheric gravity signals are distributed with a region as a function of angular distance. This behaviour, the coherence scale of pressure, the time and spatial scales appropriate to hydrostatic approximation, and the distance of the station to the ocean, suggest a division of the globe into local, regional and global zones. It is also found that the 90% atmospheric gravity signals are induced by the change in pressure in a range of 0.5° near the gravity station (the atmospheric gravity admittance is −0.360 3 μGal/hPa). It coincides rather well with that obtained when using regional distributed meteorological data sets and when analysing observed tidal gravity residuals recorded by superconducting gravimeters. The correction of the air pressure on tidal gravity shows that most of signals in observed residuals are influenced mainly by the air pressure, it is much more effective in the period of days and weeks.

The numerical results show that the computation accuracy of the column AGGF values can assure better than 1% at all the angular distances when taking an integration truncation height as 60 km above the surface of the earth. It is necessary to make correction of the height and topography at and around the gravity station based on real condition when the column AGGF values are being applied. The atmospheric gravity signal admittance is 0.059 μGal/hPa at the regional zone with an angular distance between 0.5° and 10°. The total mass is the main contributor to station gravity when the angular distance is greater than 10°. The influence of the details on various atmospheric mass distributions on the column AGGF values is not important. The atmosphere will expand and contract due to the rising and falling surface temperature. As the atmosphere warms, the center of the mass of the local column air moves further from gravity station, thus the downward positive gravity signals increases. As most of the earth's surface in the regional zone is located under instrument horizon, the effect of the change in temperature on gravity can be neglected when angular distance is greater than 10°. The atmospheric gravity admittance due to the change in temperature at local and regional zones is $0.014\ (T_0 - 15^\circ)$ μGal/℃.

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